Syllabus for MATH 3175, Group Theory

Northeastern University, Summer 2 2017

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Office hours: Tuesdays, Wednesdays, and Thursdays: 10.15am – 11.15am.

Meeting times and location: MTWR, 1.30pm – 3.10pm at Richards Hall 241.

Textbook: Abstract Algebra, 3rd edition, by John A. Beachy and William D. Blair, 2006, Waveland Press, Inc (Long Grove Illinois), ISBN 13 digit 978-1-57766-443-7.

Focus: Chapters 3,7, and some topics from Chapters 1,2.

Web Materials: All class announcements, material, and grades will be posted on Blackboard.

Topics: The course introduces the basic ideas of group theory, including symmetry groups, abelian, cyclic, and permutation groups. Also subgroups, normal subgroups, group homomorphisms, quotient groups, direct products, group actions on a set, and the Sylow theorems. We will cover parts of preparatory chapters 1-2, the chapter 3 and 7 of the text.

Goals: A. Students will understand the basic ideas and some applications of groups. Students will be able to explain groups and factor groups and their relation to symmetry. Students will recognize mathematical objects that are groups, and be able to classify them as abelian, cyclic, direct products, etc. Students will understand homomorphisms and quotients of groups, as well as group actions on a set, orbits and stabilizers, conjugacy, and be able to determine when a group has a normal subgroup.

    B. Students will be able to reason mathematically, to write simple proofs, and are able to judge when an attempted proof in group theory is correct/complete or is not.

    C. You will have a chance to reflect on doing mathematics, solving problems and on our role and progress as mathematicians

Homework: Homework will be assigned, but will NOT be collected.

Tests: There will be 4 in-class tests during the semester.

Test 1: July 13
Test 2: July 27
Test 3: August 8
Test 4: August 16

It is strongly advised that you do all of assigned homework since the tests will closely resemble the homework problems. No test scores will be dropped. There will be NO make-up tests.

Final exam:

There will be a cumulative final exam in this course. The final exam will be held at Snell Library 012 from 1pm to 3pm on August 21, 2017. **Check for exam schedule conflicts as soon as possible**. Only two finals at the same time or three in one day is a University recognized legitimate reason to be excused from taking the final at the scheduled time. Students with such a conflict should complete a final exam conflict form, available on the registrar's website.

Grading: The course grade will be determined as follows:
Final exam: 40%
Tests: 60%

Letter grades are determined numerically:

$A \geq 93$, $\quad 92 \geq A\text{-} \geq 90$, $\quad 89 \geq B\text{+} \geq 87$, $\quad 86 \geq B \geq 83$, $\quad 82 \geq B\text{-} \geq 80$,
$79 \geq C\text{+} \geq 77$, $\quad 76 \geq C \geq 73$, $\quad 72 \geq C\text{-} \geq 70$, $\quad 69 \geq D\text{+} \geq 67$, $\quad 66 \geq D \geq 63$,
$62 \geq D\text{-} \geq 60$, $\quad F \leq 59$

The grade I (Incomplete) will be given only if you have a good attendance record, have missed the final exam for a good reason, and otherwise are doing passing work. An incomplete is given at the discretion of the instructor.

Issues with the course/instructor: If you have issues with this course and/or instructor which you are not comfortable discussing with your instructor, you should contact the Teaching Director, Prof. Massey, at d.massey@northeastern.edu.

Academic Honesty: Collaboration on quizzes, tests and final exam is not allowed. From Student Code of Conduct (see http://www.northeastern.edu/osccr/academicintegrity): "A necessary prerequisite to the attainment of the goals of the University is maintaining complete honesty in all academic work. Students are expected to present as their own only that which is clearly their own work in tests and in any material submitted for credit. Students may not assist others in presenting work that is not their own. ... Offenders are subject to disciplinary action." For more on Academic Integrity see: http://www.northeastern.edu/registrar/courses/cat1213-univ-proc.pdf

Note the Following Dates:

July 17 Monday- last day to drop a summer 2 class without a W grade
July 17 Monday - last day to elect pass/fail for summer 2
July 21 Friday - last day to file a Final Exam Conflict Form for summer 2 classes
August 20 Sunday - last day to drop a summer 2 class with a W grade

Important:

1) Any student with a disability is encouraged to meet with the instructor during the first week of classes to discuss accommodations. The student must bring a current Memorandum of Accommodations from the Disability Resource Center (DRC).

2) If you are an athlete and have conflicts with an important class activity (quiz or final), you should let your instructor know before the end of second week of classes. You should also bring an official letter from the Office of Athletics.

3) All electronic devices (mobile phones, laptops etc.) should be turned off during class time, quizzes, and final exam.

4) It is University policy that no grade, including an incomplete, can be changed after one year. Exceptions must be authorized by the Academic Standing Committee.

5) Syllabus is subject to change**.** It is your responsibility to be aware of any changes the instructor may make to the syllabus as they are announced in class. Students are responsible for all information given when they are absent.

TRACE: Please complete the TRACE evaluations at the end of the course.

Schedule of Topics and Suggested Homework Exercises

Chapter 1. Integers:

   Sec 1.1 Divisors #4a,b,5,7,9,10,12.
   Sec 1.2 Primes #1a,c, 16, 19,21,24. In class 4,23
   Sec 1.3 Congruences #1a,19,20 In class 26
   Sec 1.4 Integers mod p #3a,b,9,19, 23*,24,25,27 In class 26.

Chapter 2: Functions

   Sec 2.1 Functions #1,3,5,6,9a-c,10,11,14,15.
   Sec. 2.2 Equivalence Relations #1,3,5,7,11 or 12.
   Sec 2.3 Permutations #1-4,7,10, 11,12. In class 13.

Chapter 3: Groups

   Sec 3.1: Defnition of a group #2, 3, 8, 17, 22,23 In Class:14,15,24
   Sec 3.2: Subgroups #1, 3, 5a,b; 7, 11, 19, 21,2,26. In class 12,14-16,27
   Sec 3.3: Examples #1, 2, 6, 7, 8, 9,14, 18(EC)
   Sec 3.4: Isomorphisms #1,2, 6, 8, 10, 17,19,20, 27, 28. In class 4,13, 26.
   Sec 3.5: Cyclic Groups #2, 5, 6, 10,19,20. In class 3,12 (without Prop 3.5.5).16
   Sec 3.6: Permutation Groups #1, 6,7,16, 17, 19, 20(EC), 23. In class 8,9,27.
   Sec 3.7: Homomorphisms #4, 5, 7, 8,12,14,18 In class 3,4,15,19
   Sec 3.8: Cosets, Normal Subgroups, Factor Groups #1, 2, 3, 5, 10, 12. In class 13,19-21.

Chapter 7: Structure of Groups

   Sec 7.1: Isomorphism Theorems, Automorphisms #4, 5, 8, 9,10,14,15. In class 1-3.
   Sec 7.2: Conjugacy #1, 2, 3, 7,8,9,10,13,17, 18(EC) In class 5,6,12,16
   Sec 7.3: Groups Acting on Sets: #2*,3,5, 6 ,7 ,8, 9,11,12. In class 4,5,8,
   Sec 7.4: Sylow theorems #1, 4,6, 7, 10, 12, 14. In class: 8,9.
   Sec 7.5: Finite Abelian Groups #1, 2, 3a,b; 4a,b; 11, 12
   Sec 7.6: Solvable Groups #2, 3, 6, 7,12.
   Sec 7.7: Simple Groups #1, 2, 3b, 6

Advanced topics (if time permits).

   Sec. 7.8: Nilpotent Groups (lightly): #1,3,4.
   Sec 7.9: Semidirect Products #1-3,7(EC).
   Sec 7.10: Groups of Small Order #1,2,4.

August 21st: Final exam

Further Resources: A-C are by the authors of our text. D-G have different viewpoints and may be useful.

A. Study Guide for Beginners": extra problems, some solved, Chaps 1-3
http://www.math.niu.edu/ beachy/abstract algebra/guide/contents.html

B. Review Problems on Groups and Galois Theory": (for Chap 7)
http://www.math.niu.edu/ beachy/abstract algebra/review/review.pdf

C. Some Topics in Group Theory" (Chaps 7.8-7.10 are online)
http://www.math.niu.edu/ beachy/abstract algebra/supplement.pdf

D. Frederick M. Goodman: \Algebra: Abstract and Concrete"
http://homepage.divms.uiowa.edu/ goodman/algebrabook.dir/book.2.6.pdf

E. J.S. Milne: \Group Theory": http://www.jmilne.org/math/CourseNotes/GT310.pdf

F. J. L.Alperin and Rowen B. Bell: \Groups and Representations", Springer GTM #162
(good quick review and sequel text)

G. Mark A. Armstrong, \Groups and Symmetry," Springer UTM ISBN-13: 978-0387966755.
Emphasizes geometry, symmetry and matrix groups. Used as text in Fall, 2016.

H. MacTutor site: History of group theory (J. J. O'Connor and E.F. Robinson).
http://www.gap-system.org/ history/HistTopics/Development group theory.html

I. GAP: software for working with groups http://www.gap-system.org/sitemap.html
(Prof. Gene Cooperman of CCIS at NU is one of the many authors. He and D. Kunkle wrote a program
for fast solutions of Rubik's cube http://www.ccs.neu.edu/home/kunkle/papers/kunkle-issac07.pdf).

Applications: Students have requested information about applications of group theory to other fields. Here
is a brief note by Keith Conrad (U. Conn.): http://www.math.uconn.edu/ kconrad/math216/whygroups.html
Here are a few references:

J. Anthony Zee, \Group Theory in a Nutshell for Physicists" (Princeton Univ. Press, 2016)

K. \Applications of Group Theory to Chemistry," rather nice illustrated article at LibreText site
http://chem.libretexts.org/Core/Physical and Theoretical Chemistry/Group Theory/Group Theory%3A Theory

L. George Du ey, \Applied Group Theory: For Physicists and Chemists" (Dover Books on Physics),
ISBN-13: 978-0486783147.

M. Hikmat S. Hilal and Abed Al-Hafez Sayda, \An Introductory Course on Group Theory and
Chemical Applications",(An-Najah N. University, West Bank,Palestine, 2011) ISBN: 978-1-61761-
923-6.
Meant to give access to applications without the proofs.
https://www.novapublishers.com/catalog/product info.php?products id=18082

N. Alphose Zingoni \Vibration Analysis and Structural Dynamics for Civil Engineers: Essentials
and Group-Theoretic Formulations" CRC Press (2014) ISBN 9780415522564.